2-soft-gluon exchange and factorization breaking

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A previous counterexample to disprove k_T-factorization for $H_1 + H_2 \to H_3 + H_4 + X$ is extended calculationally to one higher order in gluon exchange. The result is that, by explicit calculation, standard k_T-factorization fails for the unpolarized cross-section for the production of hadrons of high transverse momentum in hadron-hadron collisions.

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I. INTRODUCTION

Hard-scattering factorization — both conventional collinear factorization and k_T-factorization — is of great phenomenological importance in QCD phenomenology. It is therefore very important that it has been found, by the Amsterdam group [1–4], that parton densities appear to be non-universal and process-dependent for the production of high transverse momentum hadrons in hadron-hadron collisions: $H_1 + H_2 \to H_3 + H_4 + X$. This is for the case that the detected hadrons are close to back-to-back azimuthally, so that k_T-factorization, and hence transverse-momentum-dependent (TMD) parton densities and fragmentation functions, are to be used.

The changes in the parton densities involve unusual paths for the Wilson lines in their operator definitions. Although the use of these paths is quite natural, it was not completely obvious that, for example, standard factorization could not also be valid, with some nontrivial transformation relating the different kinds of TMD functions. So recently, Collins and Qiu [5] constructed a counterexample simple enough to show that this could not be the case. The simplicity of the counterexample was partly due to its application to a transverse single-spin asymmetry (SSA).

Meanwhile another approach, by Qiu, Vogelsang and Yuan, culminating in Refs. [6, 7], led to an apparently opposite result. This was that standard factorization could be valid for the SSA provided that the hard scattering factor is redefined. The contradiction is particularly striking because the model formulated in [5] as a counterexample to factorization is one to which the arguments of [6, 7] in favor of factorization also clearly apply.

Therefore it is the purpose of this paper to lay to rest any doubts about nonfactorization by extending the counterexample of [5] to one higher order of perturbation theory. The methods of [5] enable this calculation to be done quite simply.

First, certain terminological issues need to be addressed. In [5], as in the present paper, "factorization" means "standard factorization". That is, the nonperturbative parton densities and fragmentation functions for the $H_1 + H_2 \to H_3 + H_4 + X$ process either are those extracted from e^+e^- annihilation and from reactions in deep-inelastic scattering (DIS), or are related to them by purely perturbatively-based calculations. This is important for phenomenology, since perturbative calculations of hard-scattering coefficients then give predictions from first principles, to a useful degree of accuracy. In contrast, Refs. [1–4] find a more general factorization with a greater variety of reaction-dependent nonperturbative functions. Such a factorization is not standard factorization in the sense just defined.

Notice that the notorious reversal of sign of the Sivers function between DIS and Drell-Yan (DY) processes [8] is already pushing the limits of what can be accommodated under this definition of standard factorization: the actual operator definitions of the parton densities for the two processes are definitively different, and are only related numerically because of the time-reversal symmetry of QCD.

The key technical issue is that, at the level of *individual* Feynman graphs, there are extra leading-power exchanges of gluons between the subgraphs that correspond to the different factors in a statement of factorization. Factorization only holds after application of appropriate approximations followed by application of Ward identities to extract the extra gluons in particular kinematic regions from their attachments to the interior of subgraphs for other kinematic regions. Thus only after a sum over graphs can one obtain factorization, provided that the operator definitions of the parton densities and fragmentation functions are equipped with suitably compatible Wilson lines. A Wilson-line operator is the exponential of its one-gluon term; thus the use of Wilson lines implies certain relations between the values of Feynman graphs with different numbers of exchanged gluons.

The necessary approximations are only valid after certain contour deformations are applied to the momentum integrals. The results of [1–4] are essentially that the pattern of initial- and final-state parton lines in the process $H_1 + H_2 \to H_3 + H_4 + X$ is appropriate to contour deformations different from those appropriate to standard factorization. In [5], particular graphs for the single-spin asymmetry (SSA) were calculated, and led to a result that is inconsistent with standard factorization. The factor for one parton density depends on the color charge

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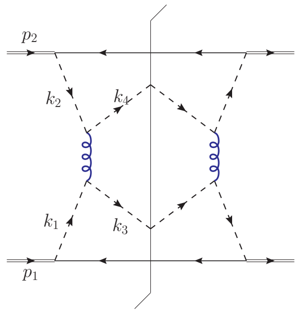


FIG. 1: (Color online.) Lowest order graph in the model for hadroproduction of hadrons of high transverse momentum. The initial state particles are color-singlet Dirac particles. The spectator lines are for Dirac "quark" fields of charges g_1 and g_2, and the active partons are for scalar "diquark" fields. The exchanged gluon line is thickened to denote the hard scattering.

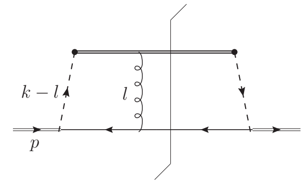


FIG. 2: Virtual one-gluon-exchange correction to parton density. The upper double line is the Wilson line, and the graph shown, together with its Hermitian conjugate gives the first contribution to the Sivers function.

of the other parton(s) participating in the process. Any factorization must be of the more general form found in [1–4], where the nonperturbative physics must be contained in extra functions defined with more complicated Wilson lines.

I will first explain, in Sec. II, how to reconcile this with the apparently opposing conclusions derived from compatible Feynman graph calculations in [6, 7]. That this discussion is rather abstract, motivates the calculations I present in Sec. III. There I calculate the effects of the exchange of an extra gluon as compared with the calculations in [5]. I will find an explicit failure of the Wilson line exponentiation for the unpolarized cross section. I will also point out that a corresponding result for the SSA will need yet an extra exchanged gluon.

II. COMPARISON WITH [6, 7]

In this section, we examine the relation between the result of nonfactorization in [5] and the argument compatible with factorization in [6, 7], which appears to encompass the model calculations in [5].

A. Direct comparison

Recall that in the model of [5], the lowest-order graph for the process $H_1 + H_2 \to H_3 + H_4 + X$ is Fig. 1, with a single hard gluon exchange. To obtain the putative Wilson-line contribution to the parton density in the lower hadron with one virtual gluon connecting the Wilson line to the spectator line, Fig. 2, we need to sum the graphs of Fig. 3. For the *real* part of the amplitude, the result does indeed correspond to standard factorization; i.e., the summed one-gluon correction in the cross section corresponds to the one-gluon correction to the parton density Fig. 2.

However for the *imaginary* part of the amplitude, there is a mismatch by a factor that depends on the colors of the active partons for the hard scattering. Thus there is a failure of the steps used in proving standard factorization: a combination of contour deformation, approximation, and a Ward identity. The imaginary part of the amplitude gives the lowest order SSA, so the effect at this order is manifested in the SSA but not in the unpolarized cross section. *If one restricts attention to this order in the number of exchanged gluons*, then, as observed in [6, 7], one obtains factorization for the SSA simply by multiplying the hard scattering by the appropriate color factor. The calculated hard scattering coefficient is different between the SSA and the unpolarized cross section.

However if standard factorization were true, graphs with more gluon exchanges would have to organize themselves into an exponentiated Wilson line operator with the color charge appropriate to the struck parton, and with the calculated color factor from Fig. 3 being the same for all the higher-order terms.

The only known argument for this is the same kind of Ward identity argument that is used in standard factorization proofs [9, 10]. The result of Collins and Qiu [5] is that the Ward identity argument fails. In contrast, Qiu, Vogelsang and Yuan [6, 7] explicitly leave the effect of extra gluon exchange to future work.

B. Importance of multiple gluon exchange

It is fairly easy to miss the central logical point of [5]. For example, Ratcliffe and Teryaev [11] state "The main point of the argument in [5] is the proportionality of the contribution of the Sivers function to the electric charge of the quark from the other (unpolarised) hadron." That much was previously quite well-known from other work, e.g., [1–4, 6, 7]. The contribution of [5] was to show in as elementary and transparent a fashion as the authors could manage that this fact must be interpreted as a breakdown of standard factorization, rather than as giving a changed normalization for the hard-scattering coefficient for the SSA. Moreover the failure of factorization is not just for the SSA, but also for the unpolarized cross section.

In view of the importance of such issues to this paper,

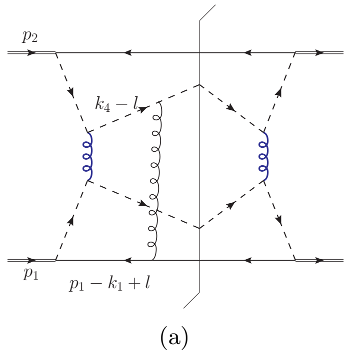


FIG. 3: (Color online.) Exchange of one extra gluon. Only graphs are shown that are relevant to the connection of the lower spectator line to the Wilson line in the associated parton density. Hermitian conjugates of these graphs also contribute, to give a total of 6 graphs.

I now re-emphasize them here.

The proof in [5] is really a meta-proof, a proof about proofs. To get factorization, one must extract, into Wilson lines, exchanges of arbitrarily many gluons between collinear and hard subgraphs. A direct calculation is evidently impractical, and any proof relies on more general methods, i.e., Ward identities. The actual calculation in [5] shows that the requisite Ward identity fails quantitatively. Therefore even though a computation at low order is compatible with factorization, the arguments needed to extend factorization to all orders do not work.

Methods for deriving factorization are general across field theories. If they work in a complicated theory like QCD, they also work in a simple model theory. Conversely and most importantly, if methods fail in the model, they will also fail in QCD. This again is a meta-proof.

Further issues are that there are many graphs of the order considered, many more beyond those actually considered, Fig. 3, and that an exact calculation of the graphs is hard. So in arguments claiming to demonstrate nonfactorization from an examination of a limited set of graphs, one has to be concerned whether some other graphs matter, or whether an inappropriate approximation was used, or whether the graphs can be interpreted differently in terms of factorization.

Therefore in [5], the model and process were carefully chosen so that only the graphs in Fig. 3 were relevant, with this being particularly clear for the SSA.

III. EXCHANGE OF EXTRA GLUONS

The model has an Abelian massive gluon, and a reaction is examined in which beam particles are color-neutral, and the partons in the lower and upper hadrons (e.g., in Fig. 1) have charges g_1 and g_2.

The lowest-order graph Fig. 1 is unambiguously consistent with factorization, with the standard lowest-order value for the hard scattering. So we consider exchanges of one or more extra gluons between the lower spectator line and the active partons k_2, k_3, k_4, as in Figs. 3, 4, and 5. These graphs are among those including the gluon exchanges whose sum must correspond to gluons attached to the Wilson in the parton density for the lower hadron, if factorization is to hold. The graphs are obtained from Fig. 1 by inserting virtual gluon lines between the lower spectator and the active partons. Note that graphs with gluons attached between the spectator and the k_1 parton are unambiguously part of the parton density.

A. Why just these graphs?

Particularly with two extra gluons, Figs. 4 and 5, there are many more graphs than those we actually examine. A priori, it is conceivable that including other graphs could change the results of the calculation, to be presented below, and hence our conclusions as to factorization or nonfactorization.

It will be necessary justify our restriction to examining just the graphs in Figs. 3, 4, and 5, together with certain related graphs. The related graphs for Fig. 3 are just Hermitian conjugates of those shown. For Figs. 4 and 5, they are those related by attaching the upper ends of the gluons in all possible ways to the active partons of the specified charge. In addition, for Fig. 5, there are the Hermitian conjugate graphs.

The following general observations will assist in the justifications of the choice of graphs.

As in [5], we use light-front coordinates where the lower hadron has large + momentum and the upper hadron has large − momentum. The initial-state and final-state poles trap the − component(s) of the momentum of the extra exchanged gluon(s) at small values, but leave the possibility of deforming the + momentum away from the poles on the active parton lines. Thus we can replace the quark propagators that join the top ends of the gluon lines to the hard scattering by eikonalized propagators, which, if factorization were to hold, would correspond directly to the Feynman rules for Wilson lines.

The general form of factorization is that the differential

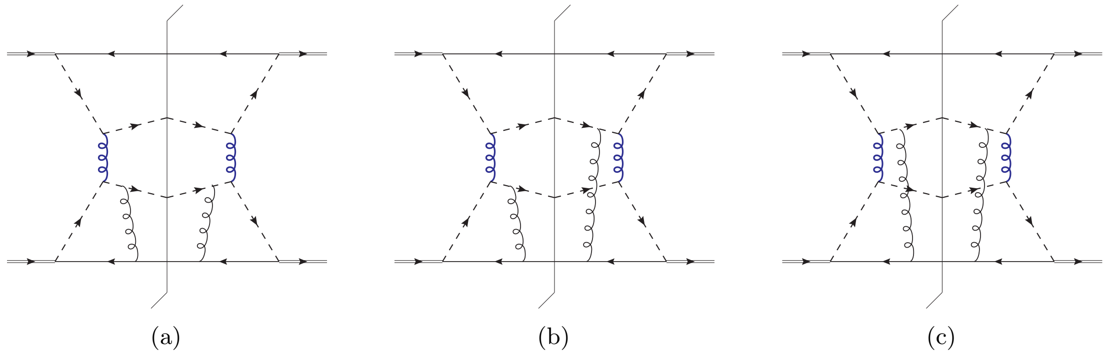


FIG. 4: (Color online.) Typical graphs for the exchange of two extra virtual gluons on opposite sides of the final-state cut, between the lower spectator and the active quark lines. The classes of graph are: (a) Two gluons attaching to the outgoing quark of charge g_1. (b) One gluon connecting to the g_1 quark, one to one of the active g_2 lines. (c) Both gluons connecting the the active g_2 lines. The total number of graphs is 9.

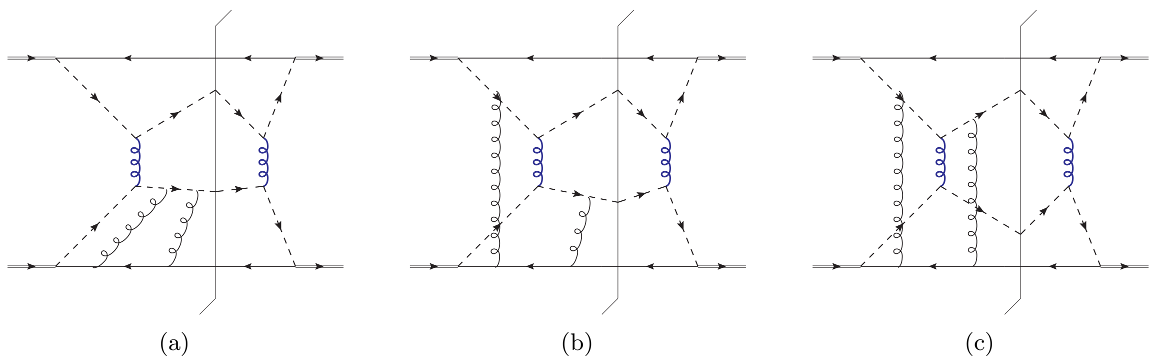


FIG. 5: (Color online.) Typical graphs for the exchange of two extra virtual gluons on one side of the final-state cut, between the lower spectator and the active quark lines. The classes of graph are: (a) Two gluons attaching to the outgoing quark of charge g_1. (b) One gluon connecting to the g_1 quark, one to one of the active g_2 lines. (c) Both gluons connecting the the active g_2 lines. The total number of graphs is 12, to which are to be added an equal number of Hermitian conjugate graphs.

cross section is a convolution

$$d\sigma = H \otimes S \otimes C_1 \otimes C_2 \otimes C_3 \otimes C_4 + \text{power-suppressed}, \quad (1)$$

with a hard factor, a soft factor, and four collinear factors that correspond to the observed particles with the same subscript numbers. Standard power-counting results lead to a number of general results for the regions that contribute to the leading power of Q, and how they would give standard factorization, were it to be valid with the known methods of proof.

Each region of a graph for the process gives a decomposition into subgraphs whose momenta correspond to the factors in Eq. (1). These subgraphs give contributions to the factors, possibly after a summation by Ward identities, possibly after some cancellations, and with the necessary subtractions to avoid double counting.

The gluon is massive, and we have chosen the high-transverse-momentum detected particles k_3 and k_4 to be almost back-to-back. Then, at leading power, the spectator lines are always collinear to their parent hadron, and thus have low transverse momentum. The active partons indicated by the dashed lines are then always collinear in the appropriate directions. In the classes of graphs shown, the extra exchanged gluons connect to the lower spectator line; these gluons can only be either soft or collinear to one of the detected initial- or final-state particles.

Here "soft" means soft the sense of [10]: all the momentum components are much less than the hard scale Q. Moreover, under the conditions stated, including a nonzero gluon mass, soft gluon lines connected to a spectator line are actually in the Glauber region (as defined in [9, 10, 12]). That is, the longitudinal components of gluon momentum are much less than the transverse gluon momentum.

What makes the Glauber region central to issues of factorization against nonfactorization is that the Ward identity arguments used to obtain factorization do not apply in the Glauber region. There are two ways in which factorization can be nevertheless obtained. One is by a contour deformation out of the Glauber region, to a collinear region or to a non-Glauber part of the soft region. The other is by a cancellation. Note that real gluons are never Glauber. Thus we restrict our attention to virtual gluon contributions.

B. Review: One extra gluon

For one extra gluon [5], we need the graphs of Fig. 3. Graphs with a virtual gluon connecting the upper spectator line to the active partons are treated exactly similarly. Graphs with a virtual gluon between the two spectators cancel after a sum over cuts, as in factorization for the Drell-Yan process. The cancellation is between final states where the spectators have different transverse momenta; thus it only occurs because the cross section is fully inclusive in the two beam-fragmentation regions. A corresponding cancellation by sum over cuts does not apply to graphs in Fig. 3, because the relative transverse momentum of the lines k_3 and k_4 is *not* integrated over.

Up to an overall factor, the same for all three graphs, Fig. 3 gives

$$\begin{aligned} E(l) &= \frac{g_2}{l^+ + i\epsilon} + \frac{g_2}{-l^+ + i\epsilon} + \frac{g_1}{-l^+ + i\epsilon} \\ &= -2\pi i g_2 \delta(l^+) + \frac{g_1}{-l^+ + i\epsilon} \\ &= -\pi i (2g_2 + g_1)\delta(l^+) - \mathrm{PV}\frac{g_1}{l^+}. \end{aligned} \tag{2}$$

For the unpolarized cross section, only the real part is relevant, i.e., the principal value in the last line. It corresponds exactly to the standard Wilson line, Fig. 2, with color charge g_1.

For the SSA we need only the imaginary part, from the delta-function term. Its coefficient is $2g_2 + g_1$, whereas the standard Wilson line in a standard parton density corresponds to a coefficient g_1, with a sign depending on the direction of the Wilson line. This is incompatible with the structure required by standard factorization.

But the lowest order contribution to the SSA, from Fig. 1, is zero. So, if one were to *ignore the possibility of exchanging even more gluons*, one might propose that factorization works but with the hard scattering multiplied by a process-dependent color factor.

C. Two extra gluons on opposite sides of final-state cut

Next, we examine the case of two extra virtual gluons, of momenta l_1 and l_2. A generalization of the arguments used with one extra gluon shows that we can restrict our attention to graphs where the gluons connect the lower spectator line with the k_2, k_3, and k_4 external lines of the hard scattering, in all possible ways. In the case that the extra gluons are on *opposite* sides of the final-state cut, typical graphs are shown in Fig. 4. If factorization were valid, the sum of these graphs would correspond to contributions to the parton density for hadron H_1 with two gluons connecting the spectator line to the Wilson line.

We again apply the eikonal approximation to the attachments next to the hard scattering, and the result is the integral over a common factor, the same for all the graphs in Fig. 4, and an eikonal factor, which is a one-gluon eikonal — as in Eq. (2) — times a complex conjugate eikonal

$$\begin{aligned} E(l_1)E(l_2)^* = g_1^2 \mathrm{PV}\frac{1}{l_1^+ l_2^+} & \\ &+ i\pi g_1(2g_2 + g_1)\delta(l_1^+)\mathrm{PV}\frac{1}{l_2^+} \\ &- i\pi g_1(2g_2 + g_1)\mathrm{PV}\frac{1}{l_1^+}\delta(l_2^+) \\ &+ \pi^2(2g_2 + g_1)^2\delta(l_1^+)\delta(l_2^+). \end{aligned} \tag{3}$$

For the SSA we need the imaginary part of this product, the middle two lines. Since these are linear in the imaginary parts of the one-gluon eikonal E, the imaginary part of the product gets the same color enhancement factor $1 + 2g_2/g_1$ as in the exchange of one extra gluon. Thus the SSA from these graphs is still consistent with the proposal that factorization holds when the hard scattering is given the color-enhancement factor.

But this is not so for the unpolarized cross section, which comes from the real part of (3), its first and last lines. The g_1^2 terms are, of course, just those that are consistent with standard factorization, and correspond to the graph in Fig. 6(a) for the parton density.

The remaining terms provide what we can call the anomaly term

$$E(l_1)E(l_2)^*|_{\mathrm{anom}} = 4\pi^2 g_2(g_2 + g_1)\delta(l_1^+)\delta(l_2^+). \tag{4}$$

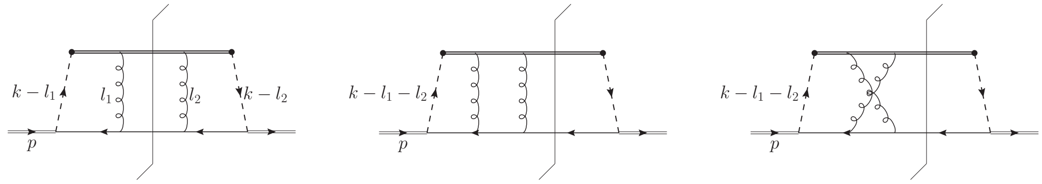
This is evidently non-zero, and corresponds to a violation of factorization for the unpolarized cross section.

However, there are still more relevant graphs that give an anomaly, those where the extra gluons are on the same side of the final-state cut. We will next analyze them, so that we can verify there is no cancellation between the different sets of graphs.

D. Two extra gluons on same side of final-state cut

Fig. 5 illustrates the graphs with two extra virtual gluons which are both on the same side of the final-state cut. Each graph is an integral over the product of a common factor and an eikonal factor. We now sum the graph-specific eikonal factors in order to find the anomaly that is to be added to the graphs in Fig. 6(b) and (c) for the parton density.

When both gluons attach to the same line, there are two graphs, and in an Abelian theory the eikonals combine in a simple way, e.g.,



(a) (b) (c)

FIG. 6: Virtual two-gluon-exchange corrections to parton density. Only graphs with gluons connecting the spectator line to the Wilson line are shown. There are two further graphs of this type which are the Hermitian conjugates of graphs (b) and (c).

$$\frac{g^2}{(-l_1^+ - l_2^+ + i\epsilon)\,(-l_1^+ + i\epsilon)} + \frac{g^2}{(-l_1^+ - l_2^+ + i\epsilon)\,(-l_2^+ + i\epsilon)} = \frac{g^2}{(-l_1^+ + i\epsilon)\,(-l_2^+ + i\epsilon)}, \tag{5}$$

where g is g_1 or g_2.

The terms proportional to g_1^2 are exemplified in Fig. 5(a), and they give exactly the result for DIS pdfs, i.e.,

$$\frac{g_1^2}{(-l_1^+ + i\epsilon)\,(-l_2^+ + i\epsilon)} = g_1^2 \mathrm{PV}\frac{1}{l_1^+ l_2^+} - \pi^2 g_1^2 \delta(l_1^+)\delta(l_2^+) + i\pi g_1^2 \delta(l_1^+)\mathrm{PV}\frac{1}{l_2^+} + i\pi g_1^2 \delta(l_2^+)\mathrm{PV}\frac{1}{l_1^+}. \tag{6}$$

The various terms proportional to g_2^2 give

$$\begin{aligned} &\frac{g_2^2}{(l_1^+ + i\epsilon)\,(l_2^+ + i\epsilon)} + \frac{g_2^2}{(l_1^+ + i\epsilon)\,(-l_2^+ + i\epsilon)} + \frac{g_2^2}{(-l_1^+ + i\epsilon)\,(l_2^+ + i\epsilon)} + \frac{g_2^2}{(-l_1^+ + i\epsilon)\,(-l_2^+ + i\epsilon)} \\ &\quad = g_2^2 \left[\frac{1}{l_1^+ + i\epsilon} + \frac{1}{-l_1^+ + i\epsilon}\right] \left[\frac{1}{l_2^+ + i\epsilon} + \frac{1}{-l_2^+ + i\epsilon}\right] \\ &\quad = -g_2^2 (2\pi)^2 \delta(l_1^+)\delta(l_2^+). \end{aligned} \tag{7}$$

This is nonzero, and contributes to the real part of the amplitude, so it clearly gives a violation of factorization for the unpolarized cross section. It can be checked that the integrals over l_1^- and l_2^- give a result that is real (and nonzero). Thus the real and imaginary parts of the eikonal correctly indicate the real and imaginary parts of the amplitude. The terms proportional to $g_1 g_2$ give

$$\begin{aligned} &\left[\frac{g_2}{l_1^+ + i\epsilon} + \frac{g_2}{-l_1^+ + i\epsilon}\right] \frac{g_1}{-l_2^+ + i\epsilon} + \left[\frac{g_2}{l_2^+ + i\epsilon} + \frac{g_2}{-l_2^+ + i\epsilon}\right] \frac{g_1}{-l_1^+ + i\epsilon} \\ &\quad = -2\pi i g_2 \delta(l_1^+) \frac{g_1}{-l_2^+ + i\epsilon} - 2\pi i g_2 \delta(l_2^+) \frac{g_1}{-l_1^+ + i\epsilon} \\ &\quad = -(2\pi)^2 g_1 g_2 \delta(l_1^+)\delta(l_2^+) + 2\pi i g_1 g_2 \delta(l_1^+)\mathrm{PV}\frac{1}{l_2^+} + 2\pi i g_1 g_2 \delta(l_2^+)\mathrm{PV}\frac{1}{l_1^+}. \end{aligned} \tag{8}$$

Notice that the total of these is just the product of two one-gluon eikonals $E(l_1)E(l_2)$. This corresponds to exponentiation in a Wilson-line operator, with the non-standard one-gluon term $E(l)$. Thus we have verified factorization in the generalized sense of [1–4]. There modified paths for the Wilson line operators are used instead of the standard ones.

As with the previous case, the imaginary part

$$g_1^2(1 + 2g_2/g_1) \left[i\pi\delta(l_1^+)\mathrm{PV}\frac{1}{l_2^+} + i\pi\delta(l_2^+)\mathrm{PV}\frac{1}{l_1^+}\right] \tag{9}$$

is linear in the anomalous part of the one-gluon eikonal. Thus it continues to be the standard imaginary part times the lowest-order color-enhancement factor for the SSA. So *at this order* we still have consistency with the factorization proposed by Qiu, Vogelsang and Yuan [6, 7] for the SSA. However, this is misleading as regards the status of factorization for the SSA. Only with yet one more gluon will the iterated anomalous imaginary part affect the SSA.

In contrast, for the unpolarized cross section, we have a high enough order to get a real contribution from the iterated anomalous one-loop eikonal. It is easily checked that the anomalous term is the negative of the anomalous

term (4) for the case that the extra gluons are on opposite sides of the final-state cut.

E. Total

We have two sets of graphs, each of which evidently gives a nonzero anomalous contribution, i.e., anomalous with respect to standard factorization. Our final step is to verify that there is no cancellation. We will also verify that we do get a cancellation if the transverse momentum of the active parton k_1 is integrated over, to correspond to the ordinary integrated parton density.

Thus we have factorization violation when TMD densities are used, but, at least at this order, we continue to have collinear factorization, provided that we work with a cross section that is not sensitive to partonic k_T.

There are two common factors for every graph considered: these are the hard scattering and the upper part of the graphs, which corresponds to the parton density in the upper hadron, both the same as in the lowest order graph, Fig. 1. The anomalous terms all correspond to graphs for the lower parton density of the form of Fig. 6, but with the Wilson line factors replaced by the relevant anomalous eikonal.

The precise values of the graphs depend on the dynamics of the theory, but to demonstrate that no cancellation follows from general principles, it is sufficient to verify non-cancellation in a simple model. Since we are no longer concerned with an SSA, we choose a simpler model than in [5]: We let all the lines that model quarks and hadrons be scalars, with a hadron-quark-quark coupling of λ. We let m_g and m_q be the gluon and quark masses, and we further simplify the kinematics by setting the hadron mass M to zero.

When the extra gluons are on opposite sides of the final-state cut, as in Fig. 6(a), the anomalous term is

$$
\begin{aligned}
I_1(k_T) = {} & \frac{\lambda^2 g_1^2 g_2 (g_2+g_1)}{(2\pi)^{12}} x p^+ \int dk^-\, d^4 l_1\, d^4 l_2\, \frac{[2(p^+-k^+)+l_1^+]\,[2(p^+-k^+)+l_2^+]}{(l_1^2-m_g^2)\,(l_2^2-m_g^2)\,[(k-l_1)-m_q^2+i\epsilon]\,[(k-l_2)-m_q^2+i\epsilon]} \times \\
& \times \frac{(2\pi)^2 \delta(l_1^+)\delta(l_2^+)\; 2\pi\delta\big((p-k)^2-m_q^2\big)}{[(p-k+l_1)-m_q^2+i\epsilon]\,[(p-k+l_2)-m_q^2+i\epsilon]} \\
= {} & \frac{\lambda^2 g_1^2 g_2 (g_2+g_1) x(1-x)}{256\pi^7} \int d^2 l_{1T}\, d^2 l_{2T} \prod_{j=1,2} \frac{k_T^2+m_q^2}{(l_{jT}^2+m_g^2)\,[(k_T-l_{jT})^2+m_q^2]}.
\end{aligned}
\tag{10}
$$

The factor xp^+ in the first line is from the definition of a parton density for a scalar quark. When the extra gluons are on the same side of the final-state cut, Fig. 6(b), (c), and their Hermitian conjugates, the anomalous term is similarly

$$
I_2(k_T) = \frac{-\lambda^2 g_1^2 g_2 (g_2+g_1) x(1-x)}{256\pi^7} \int d^2 l_{1T}\, d^2 l_{2T}\, \frac{1}{(l_{1T}^2+m_g^2)\,(l_{2T}^2+m_g^2)\,[(k_T-l_{1T}-l_{2T})^2+m_q^2]\,(k_T^2+m_q^2)}. \tag{11}
$$

Integrating $I_1(k_T)+I_2(k_T)$ over all k_T gives zero. Thus, certainly at this order, the factorization anomaly cancels in quantities that are not sensitive to partonic k_T and so can use integrated parton densities.

To verify that the cancellation is not point-by-point in k_T, we simply verify that $I_1(k_T)+I_2(k_T)$ is nonzero for one value of k_T. For example, with a certain amount of effort, it can be proved analytically that $I_1(0)+I_2(0)<0$.

IV. CONCLUSIONS

We have calculated explicitly that the graphs with two extra exchanged gluons coupling the spectator to the hard scattering give a result inconsistent with standard k_T-factorization for the unpolarized cross section. This is caused by the imaginary parts of the eikonal propagators for the partons at the hard scattering. The mismatch with standard factorization occurs because there are both initial- and final-state active partons in the process considered, production of hadrons in hadron-hadron collisions.

Any cancellation with other graphs for the parton density would require cancellations within the integral over single graphs, or between graphs for the parton density of very different topology (and hence with very different dependence on the kinematic variables). We have verified that the cancellation does not happen in a specific case.

For the imaginary part, appropriate to the SSA, the iterated anomalous imaginary part of the one-gluon eikonal does not contribute at the order of perturbation theory that we examined. So at this order there is no *explicit* contradiction with the proposal of [6, 7], where the hard scattering is modified by a color factor; such a contradiction would need a yet higher order in gluon exchange.

But it must be emphasized that the general conversion of extra gluon exchanges to the Wilson line form uses standard Ward identities. These are of a form that does *not* give the modified hard scattering. Therefore, as explained in [5], the extra color factor at one-gluon order is by itself sufficient to show that the conversion of extra gluon exchanges to standard Wilson lines fails.

Since the regions of momentum investigated are appropriate to nonperturbative physics, they correspond to a strong effective coupling in QCD. Thus the fact that nonfactorization occurs two orders of perturbation theory beyond the lowest order for a process is not indicative of any special suppression.

We have verified that at least within our example a cancellation of the anomalous term does occur if partonic k_T is integrated over. This verifies that collinear factorization continues to be valid. However, resummation methods that handle the back-to-back region are endangered, as are any other methods that are sensitive to the detailed transverse structure of the final state.

There will presumably be some further non-factorization effects when spectator-spectator interactions are included. Although such interactions cancel in the Drell-Yan process [9, 10], the results here show that the conditions for the cancellation may no longer occur when cross sections to hadrons are examined.

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